Tutorial 7. Oct 26

1. Let $I(r) = \int_\gamma \frac{e^{iz}}{z}\,dz$ where $\gamma : [0,\pi] \to \mathbb{C}$ is defined by $\gamma(t) = re^{it}$ with $r>0$. Show that $\lim_{r\to+\infty} I(r) = 0$

2. Suppose f is continuous in a region Ω. Prove that any two primitives of f (if they exist) differ by a constant.

3. Show that for $|z|<1$, one has

$$\frac{z}{1-z^2} + \frac{z^2}{1-z^4} + \cdots + \frac{z^{2^n}}{1-z^{2^{n+1}}} + \cdots = \frac{z}{1-z}$$

and

$$\frac{z}{1+z} + \frac{2z^2}{1+z^2} + \cdots + \frac{2^k z^{2^k}}{1+z^{2k}} + \cdots = \frac{z}{1-z}.$$

Hint: $$\frac{z^{2^n}}{1-z^{2^{n+1}}} = \frac{z^{2^n}+1-1}{1-z^{2^{n+1}}} = \frac{1}{1-z^{2^n}} - \frac{1}{1-z^{2^{n+1}}}$$

$$\frac{2^k z^{2^k}}{1+z^{2^k}} = \frac{-2^{k+1} z^{2^{k+1}}}{1-z^{2^{k+1}}} + \frac{2^k z^{2^k}}{1-z^{2^k}}$$